

# QCD strings ending on domain walls — a complete wetting phenomenon in SUSY QCD

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In the context of M-theory, Witten has argued that an intriguing phenomenon occurs, namely that QCD strings can end on domain walls. We present a simpler explanation of this effect using effective field theory to describe the behavior of the Polyakov loop and the gluino condensate in  $\mathcal{N} = 1$  supersymmetric QCD. We describe how domain walls separating distinct confined phases appear in this effective theory and how these interfaces are completely wet by a film of deconfined phase at the high-temperature phase transition. This gives the Polyakov loop a non-zero expectation value on the domain wall. Consequently, a static test quark which is close to the interface has a finite free energy and the string emanating from it can end on the wall.

## 1. SYMMETRIES

Witten's argument that a QCD string can end on a domain wall is made in the context of M-theory [1]. He shows that this can occur in a low energy limit of a theory in the universality class of  $\mathcal{N} = 1$  supersymmetric (SUSY)  $SU(N)$  Yang-Mills theory. This is a theory which contains gluons and their supersymmetric partners, gluinos, all of which transform under the adjoint representation of the gauge group. However, the concepts of QCD strings and domain walls naturally occur in field theory and it seems natural to explore if a field theoretical explanation of this effect could be found. There has been much investigation into domain walls in SUSY Yang-Mills theories at zero temperature [2]. We begin with  $\mathcal{N} = 1$  SUSY  $SU(N)$  Yang-Mills theory, extract the features which we think are relevant to this phenomenon and construct an effective theory at non-zero temperature. We will want to know if our effective theory contains QCD strings, which is essentially equivalent to whether or not fundamental charges are confined. From the microscopic theory, we can construct the Polyakov loop  $\Phi(\vec{x})$  whose expectation value gives us the

free energy  $F$  of a static test quark placed in our system, i.e.  $\langle \Phi \rangle \propto \exp(-F/T)$ , where  $T$  is the temperature of the system. If the quark is confined and  $F$  diverges, then  $\langle \Phi \rangle$  vanishes. Alternatively, if  $\langle \Phi \rangle$  is non-zero, then  $F$  is finite and the quark is deconfined. The microscopic theory has at non-zero temperature a discrete symmetry called the  $\mathbf{Z}(N)_c$  center symmetry, which is due to gauge transformations being not quite periodic in the Euclidean time direction. A non-zero value for the Polyakov loop, the signal for deconfinement, spontaneously breaks this center symmetry. For domain walls to occur, a global discrete symmetry must be spontaneously broken. The microscopic SUSY Yang-Mills theory also contains a discrete  $\mathbf{Z}(N)_\chi$  chiral symmetry, which is a remnant of the anomalously broken  $U(1)$  axial symmetry. This discrete chiral symmetry of the full quantum theory is broken by a non-zero expectation value for the gluino condensate  $\chi$ . If this symmetry is spontaneously broken, domain walls appear which separate regions where the system has fallen into one of the  $N$  distinct vacua, distinguished by different phases for the complex-valued gluino condensate. Our effective field theory contains the Polyakov loop and the gluino condensate and satisfies the  $\mathbf{Z}(N)_c$  center and  $\mathbf{Z}(N)_\chi$  chiral symmetries and is also invariant under charge conjugation.

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## 2. EFFECTIVE THEORY

We construct an effective theory for supersymmetric Yang-Mills theory with the gauge group  $SU(3)$  with the effective action

$$S[\Phi, \chi] = \int d^3x \left[ \frac{1}{2} |\partial_i \Phi|^2 + \frac{1}{2} |\partial_i \chi|^2 + V(\Phi, \chi) \right].$$

We demand that the potential  $V$  be invariant under  $\mathbf{Z}(3)_c$  center,  $\mathbf{Z}(3)_\chi$  chiral and charge conjugation transformations. Under  $\mathbf{Z}(3)_c$  transformations,  $\Phi \rightarrow \Phi z$  and  $\chi \rightarrow \chi$ , where  $z \in \mathbf{Z}(3)_c = \{e^{2\pi i n/3}, n = 0, 1, 2\}$ . Under  $\mathbf{Z}(3)_\chi$  transformations,  $\chi \rightarrow \chi z'$  and  $\Phi \rightarrow \Phi$ , where  $z' \in \mathbf{Z}(3)_\chi = \{e^{2\pi i m/3}, m = 0, 1, 2\}$ . Under charge conjugation,  $\Phi \rightarrow \Phi^*$  and  $\chi \rightarrow \chi^*$ . We are interested in the universal properties of all potentials  $V$  which obey these symmetries. We do not need to calculate the exact potential which is obtained by integrating out all degrees of freedom other than  $\Phi$  and  $\chi$ . The simplest potential we can write down that has these properties is

$$V(\Phi, \chi) = a|\Phi|^2 + b\Phi_1(\Phi_1^2 - 3\Phi_2^2) + c|\Phi|^4 + d|\chi|^2 + e\chi_1(\chi_1^2 - 3\chi_2^2) + f|\chi|^4 + g|\Phi|^2|\chi|^2,$$

where  $\Phi = \Phi_1 + i\Phi_2$  and  $\chi = \chi_1 + i\chi_2$ . At low temperatures, we know that the gluons and gluinos are confined and that the discrete chiral symmetry is broken. At high temperatures, the center symmetry is spontaneously broken, signalling deconfinement, and the chiral symmetry is restored. We assume that there is one first order phase transition, where the  $\mathbf{Z}(3)_c$  symmetry is spontaneously broken and simultaneously the  $\mathbf{Z}(3)_\chi$  symmetry is restored. This means that we have six types of bulk phase — three kinds of confined phase with  $\Phi^{(n)} = 0$  and  $\chi^{(n)} = \chi_0 e^{2\pi i n/3}, n = 0, 1, 2$  and three kinds of deconfined phase with  $\chi^{(n)} = 0$  and  $\Phi^{(n)} = \Phi_0 e^{2\pi i n/3}, n = 3, 4, 5$ . The temperature of the system is contained implicitly in the parameters  $a, b, \dots, g$  in the potential  $V$ . The phase transition temperature corresponds to a choice of parameters where the six bulk phases are degenerate absolute minima of the potential. We look for planar domain wall solutions of the classical equations of motion so  $\Phi(\vec{x}) = \Phi(z)$  and  $\chi(\vec{x}) = \chi(z)$ , where  $z$  is the coordinate perpen-

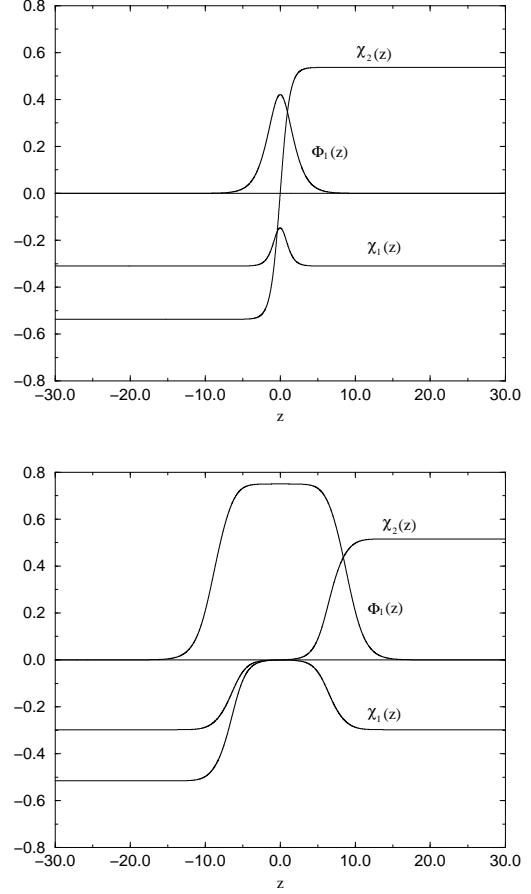


Figure 1. Profile of confined-confined domain wall (a) deep in the confined regime far from the phase transition and (b) close to the critical temperature.

dicular to the wall. The equations of motion are

$$\frac{d^2 \Phi_i}{dz^2} = \frac{\partial V}{\partial \Phi_i}, \quad \frac{d^2 \chi_i}{dz^2} = \frac{\partial V}{\partial \chi_i}.$$

We look for a numerical solution representing a domain wall separating two regions of confined bulk phase, one of type (1) and the other of type (2), i.e. we apply the boundary conditions  $\Phi(\infty) = \Phi^{(1)}, \chi(\infty) = \chi^{(1)}$  and  $\Phi(-\infty) = \Phi^{(2)}, \chi(-\infty) = \chi^{(2)}$ . In Figure 1(a), we show a solution deep in the confined regime, far from the critical temperature. However, we see that the

Polyakov loop has a non-zero value on the domain wall, showing that the interface has some of the properties of the deconfined bulk phase. In Figure 1(b), we show a solution at a temperature close to the phase transition. The confined-confined interface has now split into two confined-deconfined interfaces, which are separated by a complete wetting layer of deconfined bulk phase. Complete wetting is a universal phenomenon described by a set of critical exponents determined by the range of the interaction between the two interfaces. For example, we find that the width of the wetting layer grows logarithmically as  $z_0 \propto -\ln(T-T_c)$ , a behavior predicted for short-range forces [3]. We have found critical behavior in other quantities, e.g. the interface tension of the two confined-deconfined interfaces, and have also found an analytic solution to the equations of motion [4]. We have also investigated in detail the occurrence of complete wetting at zero temperature in systems with confined and Coulomb bulk phases [5]. Complete wetting is a very general phenomenon and was conjectured and found to occur also in non-supersymmetric  $SU(3)$  pure gauge theory [6].

However, the most important point to be extracted from the solutions is that even at a temperature deep in the confined regime, the domain wall has a non-zero Polyakov loop expectation value. This means that a static quark placed at the domain wall has a finite free energy — the infinite thin film of deconfined phase at the interface acts as a sink for the color flux emanating from the quark. Using the potential  $V$ , we can also solve for  $\Phi$  near the interface and we find that the quark's free energy  $F$  grows linearly with the distance between the quark and the domain wall. This is exactly what we would expect for a QCD string connecting the quark to the domain wall, where the energy cost is proportional to the string length. The 2-d complete wetting film can transport the color flux to infinity at a finite energy cost because of Debye screening. We assumed that deconfinement and chiral symmetry restoration occur at the same temperature. Starting at low temperature, if instead chiral symmetry is restored before deconfinement occurs, then we would again have complete wetting. However, the wetting layer would be chirally symmetric con-

fined phase and so a quark placed at the domain wall would still have an infinite free energy — a QCD string could not end there. Alternatively, if deconfinement happens before chirally symmetry restoration, we have a new phenomenon called incomplete wetting. Instead of the infinite wetting layer, bubbles of deconfined phase form at the confined-confined interface. Because these bubbles have finite volume, quantum tunneling between the various kinds of deconfined phase bubbles means that the functional integral over all domain wall solutions gives  $\langle \Phi \rangle = 0$ , so again a quark placed at the interface has an infinite free energy. This is consistent with Gauss' law which says that the string cannot end on a bubble as it has a compact surface. Quantum tunneling does not occur between infinite films of distinct types of deconfined phase as these solutions are separated by infinite energy barriers. Note that it is essential that the domain wall be infinite so that the color flux can be transported to infinity. If we believe Witten's argument that QCD strings can end on domain walls, then we can make a prediction for the phase structure of  $SU(N)$  Yang-Mills theory with gluons and gluinos. We conclude that in this model chiral symmetry is restored and deconfinement occurs at the same temperature.

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